Insights into the ABg BSDF model

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The Total Scatter for the ABg BSDF model can be computed in closed form for certain conditions as a function of A and B. This can give insight into the behavior of scattering versus these parameters in the ABg model.

When the direction of incidence is normal to the scattering surface (i.e. $\beta_0 = 0$), B > 0, and g has certain integer values, the Total Scatter integral can be computed in closed form. The Total Scatter integral for normal incidence is

$$TS = \int_0^{2\pi} \int_0^1 \frac{A}{B+\beta^g} \beta d\beta \, d\varphi,$$

where β is the radial component of the **β** vector. The φ integral is trivial, and we are left with

$$TS = 2\pi A \int_0^1 \frac{1}{B+\beta^g} \beta d\beta.$$

This integral can be solved analytically for g = 0, 1, 2, and 3 (and possibly higher integer values of g). The results are shown in the table below.

g	**ABg Total Scatter**
0	$\frac{\pi A}{B+1}$
1	$2\pi A \left(1 - B \, ln\left(\frac{B+1}{B}\right)\right)$
2	$\pi A \, ln\left(\frac{B+1}{B}\right)$
3	$2\pi A \left[-\frac{1}{3\alpha}\left\{\frac{1}{2} ln\left(\frac{(1-\alpha)^2}{1-\alpha-\alpha^2}\right) - \sqrt{3}\left[tan^{-1}\left(\frac{2-\alpha}{\alpha\sqrt{3}}\right) - tan^{-1}\left(\frac{-\alpha}{\alpha\sqrt{3}}\right)\right]\right\}\right]$, where $\alpha = \sqrt[3]{B}$