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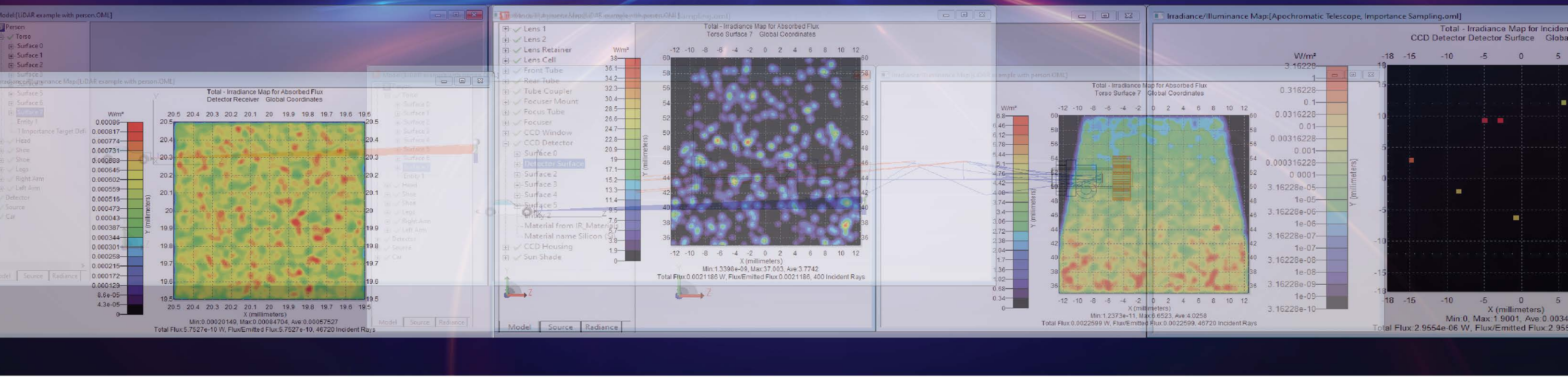
Total - Irradiance Map for Absorbed Flux
Detector Receiver Global Coordinates
Total - Irradiance Map for Absorbed Flux
Torso Surface 7 Global Coordinates
Irradiance/Illuminance Map:[Apochromatic Telescope, Importance Sampling.oml]
Total - Irradiance Map for Incident
CCD Detector Detector Surface
X (millimeters)
Y (millimeters)
W/m²
Model
Source
Radiance

Improving the Efficiency of Monte Carlo Raytracing using Importance Sampling

Dave Jacobsen

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Author Bio: Dave Jacobsen is a Senior Applications Engineer at Lambda Research Corporation with over 30 years of optical engineering experience. Dave is Lambda Research Corporation's senior sales engineer and teaches many TracePro training classes, both in the US and worldwide. Prior to coming to Lambda Research Corporation, Dave worked as a Principal Optical Engineer at PerkinElmer, formerly EG&G, working with xenon flashlamp based systems for illumination, machine vision, and process control, as well as designing spectroscopy systems for biomedical applications.

Abstract: Monte Carlo raytracing algorithms have long been used in optical design and analysis software. A limitation of the Monte Carlo method is that low probability events or ray paths may be undersampled. In this paper we will look at using Importance Sampling to improve the results in undersampled ray paths.

I. Introduction

The Monte Carlo method is a common tool in modern optical design and analysis software. The Monte Carlo method is used to simulate the scattering and diffraction of light, and to sample the distributions of rays from light sources. In Monte Carlo raytracing, scattering and diffraction are treated as random processes. Instead of propagating a distribution of light, discrete samples of the distribution, or rays, are propagated. The samples are randomly chosen, using the scattering distribution function as a probability density. This allows the well-developed techniques of raytracing to be used to model scattering[1].

In “brute force” Monte Carlo raytracing, the directions of rays are chosen randomly, and a reliable answer is obtained by tracing a very large number of rays. Variance reduction techniques can be employed to reduce the number of rays required to get an accurate result.

The fact that the Monte Carlo method uses the scattering distribution function as a probability density can lead to low probability paths being undersampled. Variance reduction techniques such as importance sampling can be used to improve the results in these undersampled paths.

II. Importance Sampling

One method of variance reduction is ray splitting. When a ray hits a surface during a raytrace, the ray can be split into one or more rays. The flux of each resulting ray is determined by the property parameters of the surface. The flux of the rays is apportioned to maintain conservation of energy.

Importance sampling is another common method of variance reduction. Importance sampling is a Monte Carlo technique in which rays are propagated in specific directions in the optical system, which are important in determining the results you need.

One way to think of importance sampling is think of it as a way to improve the likely hood of an event of interest occurring. An example of this would be rolling a pair of dice and hoping for a 3. To get a 3 we would need to roll a 1 on one die and 2 on the other. With a normal, or fair, pair of dice the likelihood of rolling a 3 is $2 * \left(\frac{1}{6}\right) * \left(\frac{1}{6}\right) = 1/18.$ To try and improve the results we could "modify" the dice so that 1s and 2s occur with probability of 1/3 instead of 1/6. Now the likelihood of rolling a 3 is 4 times as likely. This is the basis of importance sampling. Please note that this is an experiment I would only suggest trying at home.

Another example, optics related this time, would be a trying to determine the chance of a ray hitting a detector from and out of field of view source in a well-designed optical system with good stray light attenuation. In a case like this you may need to trace 10^{20} rays or more using Crude Monte Carlo techniques.

One important key factor in importance sampling is to increase the number of rays going towards a particular importance sampling target while not changing the overall flux at the target. The flux applied to the ray is calculated by integrating the BSDF (bidirectional scattering distribution function) over the solid angle subtended by the importance sampling target as seen from the intersecting point on the scattering surface, and multiplying by the incident flux. Since the flux of some importance sampled rays is underestimated and some are overestimated, it is necessary to average the flux over a large number of rays.

In some cases, the importance sampling target subtends too large a solid angle, or the BSDF of the surface varies significantly, more than an order of magnitude, over the subtended angle. In this case you need more rays to hit the target to reduce noise. To accomplish this, the sampling target can be divided into cells to create a stratified target. This stratified importance sampling is a feature in some optical analysis software programs, especially those with strong stray light analysis capabilities.

III. Applications of Importance Sampling

One of the most common applications of importance sampling is in stray light analysis. Stray light can be thought of as unwanted light that makes its way to a detector or target surface. This stray light can result in reduced performance of the system, such as decreased contrast or a lower signal-to-noise ratio. If the system is well designed the percentage of stray light that makes it to the detector will be exceeding small. In order to avoid having to trace a very large number of rays with increased ray-trace times, importance sampling can be used to get more rays to the detector with a reasonable ray-trace time.

Figure 1 shows a refracting telescope with a doublet objective. In this example we want to look at the stray light on the CCD detector due to an off-axis, out- of-field-of-view source. In this case the source is 5 degrees off-axis. The inside of the optical tube is coated with a black coating. **Figure 2** shows the results of tracing 10000 rays. Only the rays that hit the detector are shown in this image. In this example only 8 rays make it to the detector from the initial 10000 started. This may be good in terms of stray light performance, but for stray light analysis we would like to have a larger sample of rays.

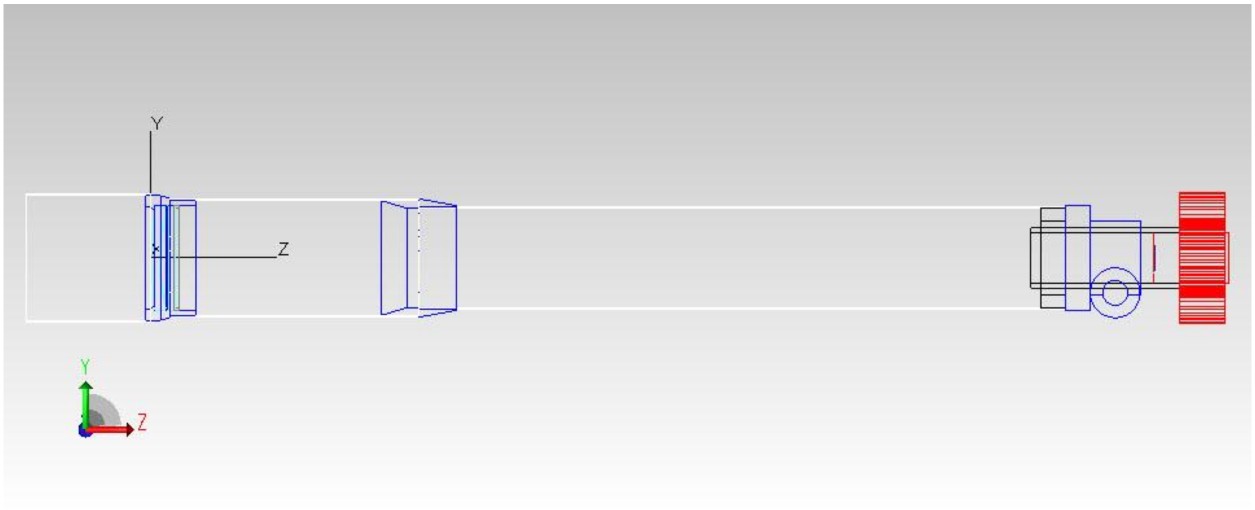
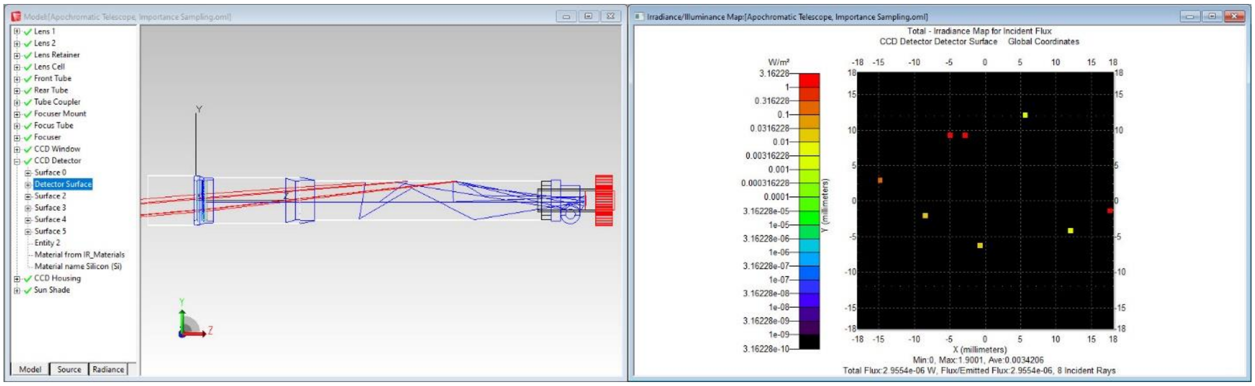


Figure 1: Telescope example


Model[Apochromatic Telescope, Importance Sampling.oml]

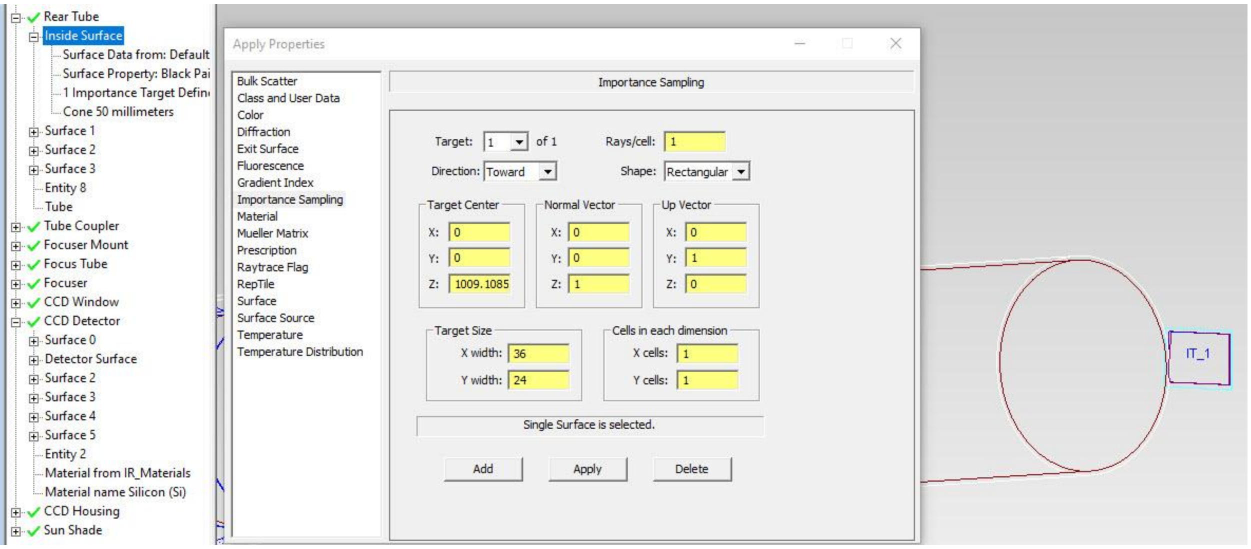
Irradiance/Illuminance Map:[Apochromatic Telescope, Importance Sampling.oml]

Total - Irradiance Map for Incident Flux
CCD Detector Detector Surface   Global Coordinates

Min:0, Max:1.9001, Ave:0.0034206
Total Flux:2.9554e-06 W, Flux/Emitted Flux:2.9554e-06, 8 Incident Rays

Figure 2: Rays from off-axis source

If we want to improve the sampling of the rays from the inside of the optical toward towards the CCD detector, we can apply an importance sampling target. This target will be from the inside surface of the optical tube towards the CCD surface. The parameters of the importance sampling target are shown in **Figure 3**. **Figure 4** shows the results after applying the importance sampling target. Now there are over 6700 rays getting to the detector, a more than 800x increase in the number of rays without increasing the number of rays started, and with only a marginal increase in raytrace time.



Importance Sampling

Target: 1 of 1 Rays/cell: 1

Direction: Toward Shape: Rectangular

Target Center X: 0 Y: 0 Z: 1009.1085

Normal Vector X: 0 Y: 0 Z: 1

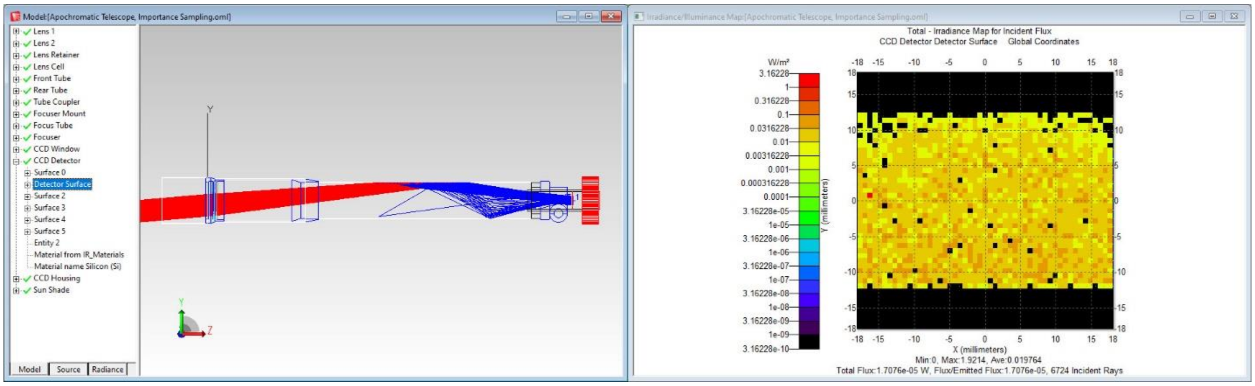
Up Vector X: 0 Y: 1 Z: 0

Target Size X width: 36 Y width: 24

Cells in each dimension X cells: 1 Y cells: 1

Single Surface is selected.

Figure 3: Importance sample target parameters



Min:0, Max:1.9214, Ave:0.019764

Total Flux:1.7076e-05 W, Flux/Emitted Flux:1.7076e-05, 6724 Incident Rays

Figure 4: Raytrace results with importance sampling applied

The uses of importance sampling are not limited to stray light applications. Other uses include non-imaging analysis using scattering, especially when the collecting area is relatively small, or when using a surface source and you want a large number of rays to be sampled in a particular, narrow direction. An example of both of these cases is LiDAR, light detection and ranging. There are currently numerous applications for LiDAR, especially in automotive applications, including autonomous driving vehicles. The output beam of the LiDAR system is usually an elliptical beam. Since this beam will spread out over distance, only a small portion of the beam may illuminate a target. It would be useful in analyzing this type of system to improve the number of rays on the target using importance sampling. The return signal to the detector will be even smaller due to scattering and distance, so importance sampling from the target to the detector will be very helpful.

Figure 5 shows a LiDAR example where a source is used to illuminate a “target,” a pedestrian in this case, and then light is scattered from the pedestrian backs towards the detector on the car. In this example 100000 rays were launched from the source in a 5 x 25-degree FWHM beam.

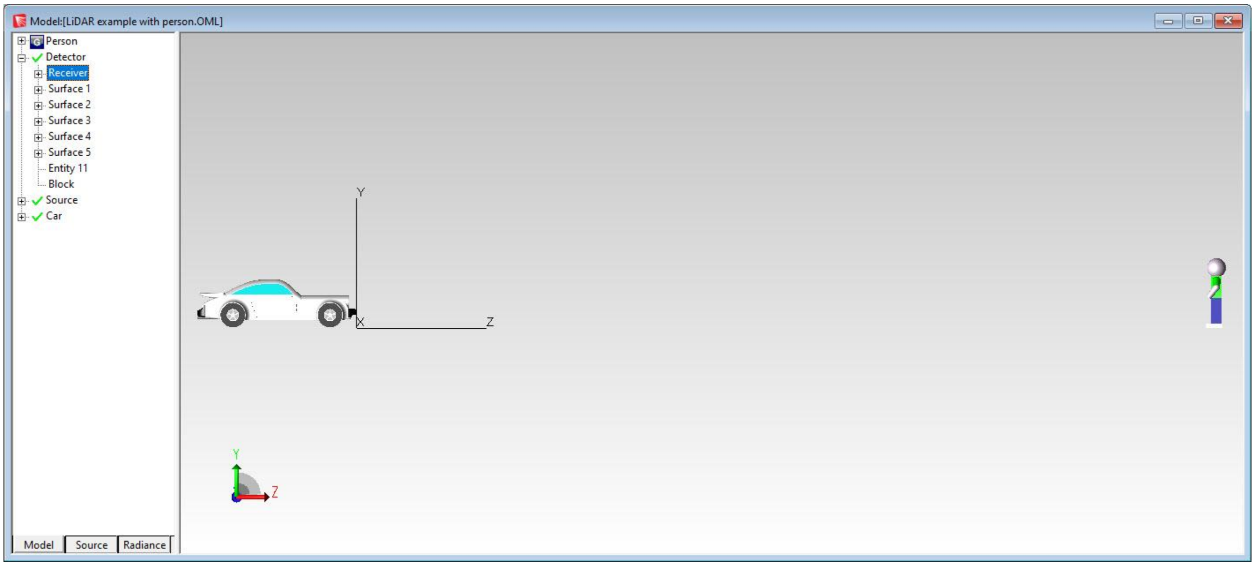
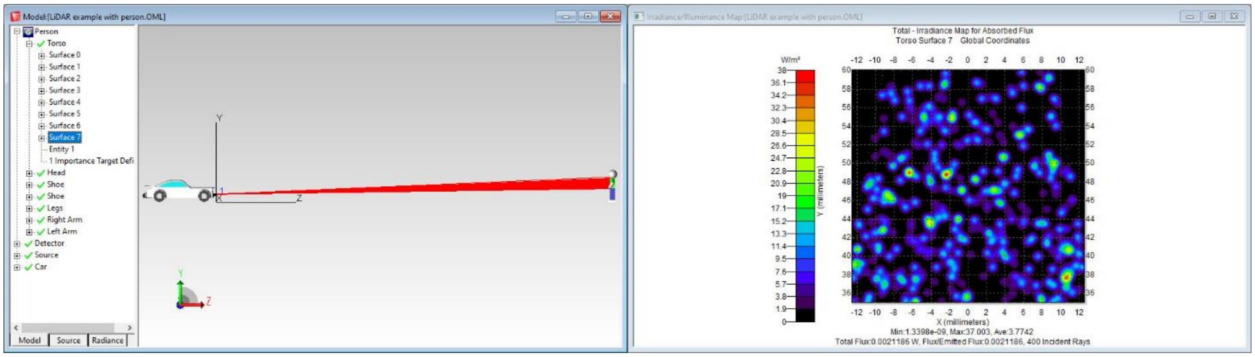


Figure 5: LiDAR example

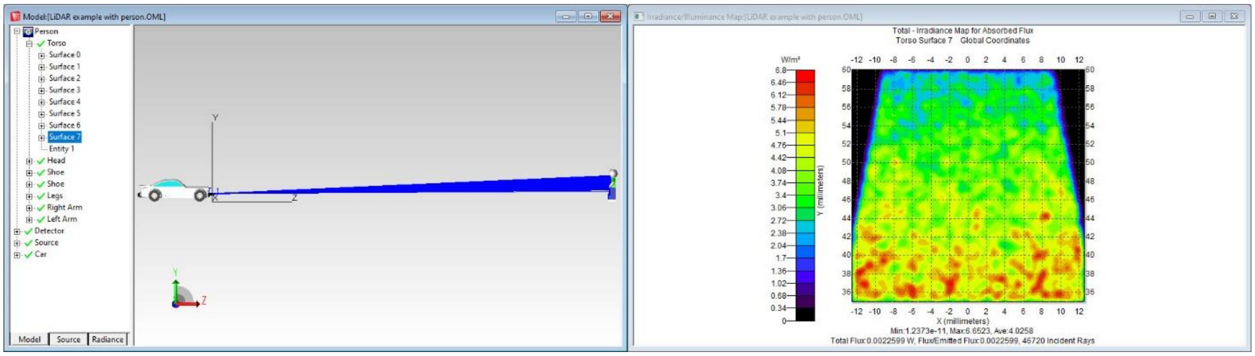
Figure 6 shows that without importance sampling only 400 of the original rays hit the torso of the pedestrian.



Min:1.3398e-09, Max:37.003, Ave:3.7742
Total Flux:0.0021186 W, Flux/Emitted Flux:0.0021186, 400 Incident Rays

Figure 6: Rays from source without importance sampling

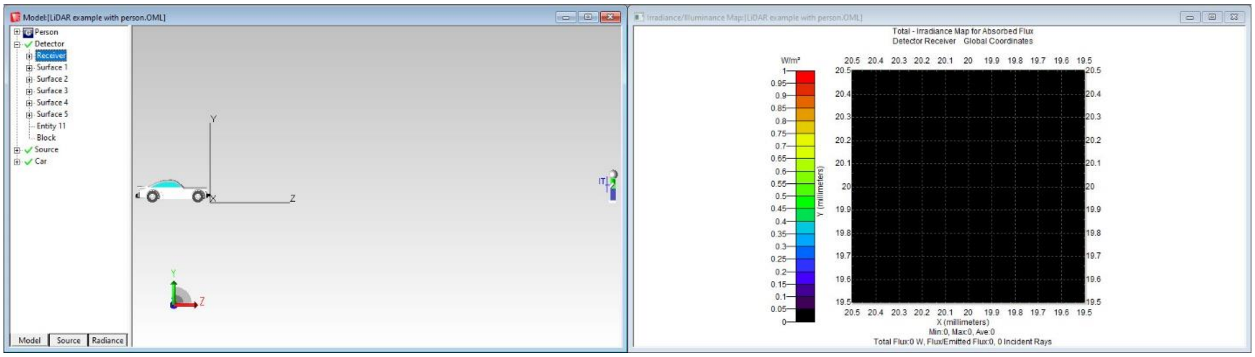
An importance sampling target was applied from the source towards the torso of the pedestrian. **Figure 7** shows that 46720 rays now strike the torso of the pedestrian.



Min:1.2373e-11, Max:6.6523, Ave:4.0258
Total Flux:0.0022599 W, Flux/Emitted Flux:0.0022599, 46720 Incident Rays

Figure 7: Rays from source with importance sampling

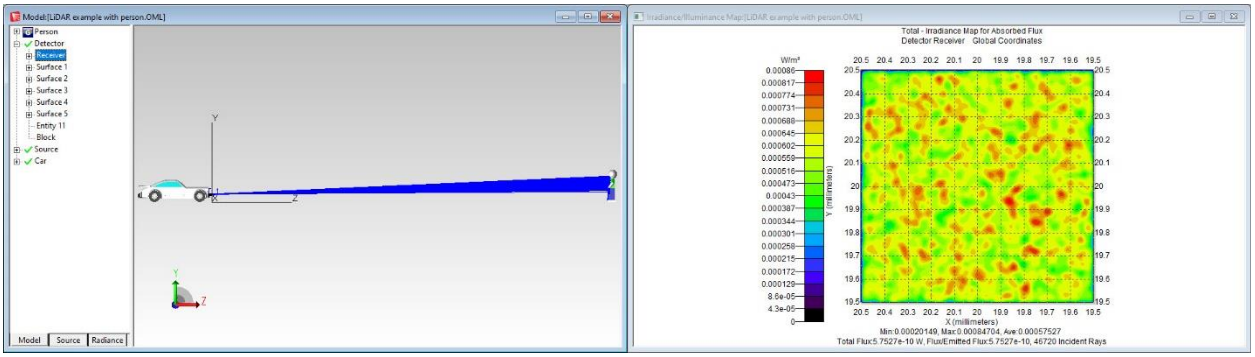
If we now look at the rays scattered back towards the detector from the pedestrian's torso, we see in **Figure 8** that no rays make it to the detector initially.



Min:0, Max:0, Ave:0
Total Flux:0 W, Flux/Emitted Flux:0, 0 Incident Rays

Figure 8: Rays at detector without importance sampling

Applying an importance sampling target from the pedestrian's torso towards the detector results in 46720 rays hitting the detector, as shown in **Figure 9**.



Min:0.00020149, Max:0.00084704, Ave:0.00057527
Total Flux:5.7527e-10 W, Flux/Emitted Flux:5.7527e-10, 46720 Incident Rays

Figure 9: Rays at detector with importance sampling

This example shows that using importance sampling can dramatically increase the efficiency of the raytrace without the need for tracing more rays to start. This increase in efficiency leads to more rays on the detector with less effort.

IV. Conclusions

The Monte Carlo method is an excellent choice for raytracing applications and allows for fast and efficient analysis of optical systems. In cases where the probability of an event is low, such as a stray light path, or a small detector with a distant highly scattering target, the Crude Monte Carlo method may have limitations. Importance sampling is a quick and easy method that allows for improved efficiency in undersampled paths without having to increase the number of rays started.

V. References

1. *TracePro User's Manual, Release 2020, Lambda Research Corporation*